

Chapter 1

THE SOLOW GROWTH MODEL

1.1 Some Basic Facts about Economic Growth

Over the past few centuries, standards of living in industrialized countries have reached levels almost unimaginable to our ancestors. Although comparisons are difficult, the best available evidence suggests that average real incomes today in the United States and Western Europe are between 10 and 30 times larger than a century ago, and between 50 and 300 times larger than two centuries ago.¹

Moreover, worldwide growth is far from constant. Growth has been rising over most of modern history. Average growth rates in the industrialized countries were higher in the twentieth century than in the nineteenth, and higher in the nineteenth than in the eighteenth. Further, average incomes on the eve of the Industrial Revolution even in the wealthiest countries were not dramatically above subsistence levels; this tells us that average growth over the millennia before the Industrial Revolution must have been very, very low.

One important exception to this general pattern of increasing growth is the *productivity growth slowdown*. Average annual growth in output per person in the United States and other industrialized countries from the early 1970s to the mid-1990s was about a percentage point below its earlier level. The data since then suggest a rebound in productivity growth, at least in the United States. How long the rebound will last and how widespread it will be are not yet clear.

¹ Maddison (2003) reports and discusses basic data on average real incomes over modern history. Most of the uncertainty about the extent of long-term growth concerns the behavior not of nominal income, but of the price indexes needed to convert those figures into estimates of real income. Adjusting for quality changes and for the introduction of new goods is conceptually and practically difficult, and conventional price indexes do not make these adjustments well. See Nordhaus (1997) and Boskin, Dulberger, Gordon, Griliches, and Jorgenson (1998) for discussions of the issues involved and analyses of the biases in conventional price indexes.

There are also enormous differences in standards of living across parts of the world. Average real incomes in such countries as the United States, Germany, and Japan appear to exceed those in such countries as Bangladesh and Kenya by a factor of about 20.² As with worldwide growth, cross-country income differences are not immutable. Growth in individual countries often differs considerably from average worldwide growth; that is, there are often large changes in countries' relative incomes.

The most striking examples of large changes in relative incomes are *growth miracles* and *growth disasters*. Growth miracles are episodes where growth in a country far exceeds the world average over an extended period, with the result that the country moves rapidly up the world income distribution. Some prominent growth miracles are Japan from the end of World War II to around 1990 and the newly industrializing countries (NICs) of East Asia—South Korea, Taiwan, Singapore, and Hong Kong—starting around 1960. Average incomes in the NICs, for example, have grown at an average annual rate of over 5 percent since 1960. As a result, their average incomes relative to that of the United States have more than tripled.

Growth disasters are episodes where a country's growth falls far short of the world average. Two very different examples of growth disasters are Argentina and many of the countries of sub-Saharan Africa. In 1900, Argentina's average income was only slightly behind those of the world's leaders, and it appeared poised to become a major industrialized country. But its growth performance over most of the twentieth century was dismal, and it is now near the middle of the world income distribution. Sub-Saharan African countries such as Chad, Ghana, and Mozambique have been extremely poor throughout their histories and have been unable to obtain any sustained growth in average incomes. As a result, their average incomes have remained close to subsistence levels while average world income has been rising steadily.

Other countries exhibit more complicated growth patterns. Cote d'Ivoire was held up as the growth model for Africa through the 1970s. From 1960 to 1978, real income per person grew at an average annual rate of 3.5 percent. But in the next decade, average income fell by a third. To take another example, average growth in Mexico was extremely high in the 1960s and 1970s, negative in most of the 1980s, and again very high—with a brief but severe interruption in the mid-1990s—since then.

Over the whole of the modern era, cross-country income differences have widened on average. The fact that average incomes in the richest countries at the beginning of the Industrial Revolution were not far above subsistence

² Comparisons of real incomes across countries are far from straightforward, but are much easier than comparisons over extended periods of time. The basic source for cross-country data on real income is the Penn World Tables. Documentation of these data and the most recent figures are available at the National Bureau of Economic Research's web site, <http://www.nber.org>.

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³ The Solow model
by Robert Solow (Solo

means that the overall dispersion of average incomes across different parts of the world must have been much smaller than it is today (Pritchett, 1997). Over the past few decades, however, there has been no strong tendency either toward continued divergence or toward convergence.

The implications of the vast differences in standards of living over time and across countries for human welfare are enormous. The differences are associated with large differences in nutrition, literacy, infant mortality, life expectancy, and other direct measures of well-being. And the welfare consequences of long-run growth swamp any possible effects of the short-run fluctuations that macroeconomics traditionally focuses on. During an average recession in the United States, for example, real income per person falls by a few percent relative to its usual path. In contrast, the productivity growth slowdown reduced real income per person in the United States by about 25 percent relative to what it otherwise would have been. Other examples are even more startling. If real income per person in Bangladesh continues to grow at its postwar average rate of 1.1 percent, it will take well over 200 years for it to reach the current U.S. level. If Bangladesh achieves 3 percent growth, the time will be reduced to 100 years. And if it achieves 5 percent growth, as the NICs have done, the process will take only 60 years. To quote Robert Lucas (1988), "Once one starts to think about [economic growth], it is hard to think about anything else."

The first three chapters of this book are therefore devoted to economic growth. We will investigate several models of growth. Although we will examine the models' mechanics in considerable detail, our goal is to learn what insights they offer concerning worldwide growth and income differences across countries. Indeed, the ultimate objective of research on economic growth is to determine whether there are possibilities for raising overall growth or bringing standards of living in poor countries closer to those in the world leaders.

This chapter focuses on the model that economists have traditionally used to study these issues, the Solow growth model.³ The Solow model is the starting point for almost all analyses of growth. Even models that depart fundamentally from Solow's are often best understood through comparison with the Solow model. Thus understanding the model is essential to understanding theories of growth.

The principal conclusion of the Solow model is that the accumulation of physical capital cannot account for either the vast growth over time in output per person or the vast geographic differences in output per person. Specifically, suppose that capital accumulation affects output through the conventional channel that capital makes a direct contribution to production, for which it is paid its marginal product. Then the Solow model implies that the differences in real incomes that we are trying to understand are far

³ The Solow model (which is sometimes known as the Solow-Swan model) was developed by Robert Solow (Solow, 1956) and T. W. Swan (Swan, 1956).

too large to be accounted for by differences in capital inputs. The model treats other potential sources of differences in real incomes as either exogenous and thus not explained by the model (in the case of technological progress, for example) or absent altogether (in the case of positive externalities from capital, for example). Thus to address the central questions of growth theory, we must move beyond the Solow model.

Chapters 2 and 3 therefore extend and modify the Solow model. Chapter 2 investigates the determinants of saving and investment. The Solow model has no optimization in it; it simply takes the saving rate as exogenous and constant. Chapter 2 presents two models that make saving endogenous and potentially time-varying. In the first, saving and consumption decisions are made by a fixed set of infinitely lived households; in the second, the decisions are made by overlapping generations of households with finite horizons.

Relaxing the Solow model's assumption of a constant saving rate has three advantages. First, and most important for studying growth, it demonstrates that the Solow model's conclusions about the central questions of growth theory do not hinge on its assumption of a fixed saving rate. Second, it allows us to consider welfare issues. A model that directly specifies relations among aggregate variables provides no way of judging whether some outcomes are better or worse than others: without individuals in the model, we cannot say whether different outcomes make individuals better or worse off. The infinite-horizon and overlapping-generations models are built up from the behavior of individuals, and therefore can be used to discuss welfare issues. Third, infinite-horizon and overlapping-generations models are used to study many issues in economics other than economic growth; thus they are valuable tools.

Chapter 3 investigates more fundamental departures from the Solow model. Its models, in contrast to Chapter 2's, provide different answers than the Solow model to the central questions of growth theory. The first part of the chapter departs from the Solow model's treatment of technological progress as exogenous; it assumes instead that it is the result of the allocation of resources to the creation of new technologies. We will investigate the implications of such *endogenous technological progress* for economic growth and the determinants of the allocation of resources to innovative activities.

The main conclusion of this analysis is that endogenous technological progress is almost surely central to worldwide growth but probably has little to do with cross-country income differences. The second part of Chapter 3 therefore focuses specifically on those differences. We will find that understanding those differences requires considering two new factors: differences in human as well as physical capital, and differences in productivity not stemming from differences in technology. This material explores both how those factors can help us understand the enormous differences

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We now turn to the Solow model.

1.2 Assumptions

Inputs and Output

The Solow model focuses on four variables: output (Y), capital (K), labor (L), and “knowledge” or the “effectiveness of labor” (A). At any time, the economy has some amounts of capital, labor, and knowledge, and these are combined to produce output. The production function takes the form

$$Y(t) = F(K(t), A(t)L(t)), \quad (1.1)$$

where t denotes time.

Notice that time does not enter the production function directly, but only through K , L , and A . That is, output changes over time only if the inputs to production change. In particular, the amount of output obtained from given quantities of capital and labor rises over time—there is technological progress—only if the amount of knowledge increases.

Notice also that A and L enter multiplicatively. AL is referred to as *effective labor*, and technological progress that enters in this fashion is known as *labor-augmenting* or *Harrod-neutral*.⁴ This way of specifying how A enters, together with the other assumptions of the model, will imply that the ratio of capital to output, K/Y , eventually settles down. In practice, capital-output ratios do not show any clear upward or downward trend over extended periods. In addition, building the model so that the ratio is eventually constant makes the analysis much simpler. Assuming that A multiplies L is therefore very convenient.

The central assumptions of the Solow model concern the properties of the production function and the evolution of the three inputs into production (capital, labor, and knowledge) over time. We discuss each in turn.

Assumptions Concerning the Production Function

The model’s critical assumption concerning the production function is that it has constant returns to scale in its two arguments, capital and effective labor. That is, doubling the quantities of capital and effective labor (for example, by doubling K and L with A held fixed) doubles the amount produced.

⁴ If knowledge enters in the form $Y = F(AK, L)$, technological progress is *capital-augmenting*. If it enters in the form $Y = AF(K, L)$, technological progress is *Hicks-neutral*.